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THERMODYNAMIC INFLUENCES ON THE BEHAVIOR OF ONE-DIMENSIONAL SHOCK WAVES IN ELASTIC DIELECTRICS

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1. INTRODUCTION

In this paper, we examine the behavior of one-dimensional shock waves propagating in elastic dielectrics. We account for the presence of thermodynamic effects but ignore the possibility of heat conduction. After deriving the differential equation which the amplitude of a shock must obey, we examine the properties of shock transition. In particular, we find that the classical results of shock transition can be generalized to the present situation; and these results, in turn, allow us to deduce the implications of the shock amplitude equation on the behavior of the shock. It is found that the criteria of whether the shock grows or decays depends on the relative magnitudes of the jump in strain gradient and λ_c, called the critical jump in strain gradient. This, of course, is expected. We also examine the properties of the temperature and electric field during shock transition and derive particular results which are valid when the shock amplitude is infinitesimal.

2. BASIC EQUATIONS AND CONSTITUTIVE ASSUMPTIONS

In this paper we consider the one-dimensional motion of an elastic dielectric body $\mathscr{B}$ neglecting the effects of heat conduction. The motion is described by a scalar-valued function χ which gives the location $x = \chi(X, t)$ at time t of the material point X. As is customary, we identify the material point X with its position in a fixed homogeneous configuration $\mathscr{R}$ with mass density ρ_0. For such a material the specific internal energy e, the stress T, the material electrical displacement $\mathscr{D}$, and the absolute temperature θ are determined by the strain ε, the material electrical field $\mathscr{E}$, and the specific entropy η:

$$\begin{aligned} e &= \hat{e}(\varepsilon, \mathscr{E}, \eta), \\ T &= \hat{T}(\varepsilon, \mathscr{E}, \eta), \\ \mathscr{D} &= \hat{\mathscr{D}}(\varepsilon, \mathscr{E}, \eta), \\ \theta &= \hat{\theta}(\varepsilon\ \mathscr{E}, \eta) \end{aligned} \tag{2.1}$$

with

$$\hat{T}(\varepsilon, \mathscr{E}, \eta) = \rho_0 \frac{\partial \hat{\Gamma}(\varepsilon, \mathscr{E}, \eta)}{\partial \varepsilon},$$
$$\hat{\mathscr{D}}(\varepsilon, \mathscr{E}, \eta) = -\rho_0 \frac{\partial \hat{\Gamma}(\varepsilon, \mathscr{E}, \eta)}{\partial \mathscr{E}}, \tag{2.2}$$
$$\hat{\theta}(\varepsilon, \mathscr{E}, \eta) = \frac{\partial \hat{\Gamma}(\varepsilon, \mathscr{E}, \eta)}{\partial \eta}.$$

In (2.2)

$$\Gamma = \hat{\Gamma}(\varepsilon, \mathscr{E}, \eta) = \hat{e}(\varepsilon, \mathscr{E}, \eta) + \frac{\varepsilon_0 \mathscr{E}^2}{2\rho_0} - \frac{\mathscr{E}\hat{\mathscr{D}}(\varepsilon, \mathscr{E}, \eta)}{\rho_0} \tag{2.3}$$

with ε_0 being the electrical permitivity of free space. The strain ε is defined by

$$\varepsilon(X, t) = \frac{\partial u(X, t)}{\partial X} \tag{2.4}$$

where $u(X, t) = \chi(X, t) - X$ is the displacement at time t of the material point X. We assume that the electromagnetic conditions in the material are quasi-static and that magnetic effects are negligible so that

$$\mathscr{E}(X, t) = -\frac{\partial \Phi(X, t)}{\partial X} \tag{2.5}$$

where $\Phi(X, t)$ is the electrical potential. We note that the actual electric field E at the point X is related to $\mathscr{E}$ by the formula

$$\mathscr{E} = (1 + \varepsilon)E. \tag{2.6}$$

In the absence of an external body force, external heat supply and free charge, the balance equations are of the form

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{X_\alpha}^{X_\beta} \rho v \,\mathrm{d}X = T(X_\beta, t) - T(X_\alpha, t),$$
$$\mathscr{D}(X_\beta, t) - \mathscr{D}(X_\alpha, t) = 0,$$
$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{X_\alpha}^{X_\beta} \left(\rho_0 e + \frac{\varepsilon_0 \mathscr{E}^2}{2} + \frac{1}{2}\rho_0 v^2\right) \mathrm{d}X = T(X_\beta, t)v(X_\beta, t) - T(X_\alpha, t)v(X_\alpha, t) \tag{2.7}$$
$$-\Phi(X_\beta, t)\dot{\mathscr{D}}(X_\beta, t) + \Phi(X_\alpha, t)\dot{\mathscr{D}}(X_\alpha, t)$$

where a superimposed dot denotes the material time derivative, and $v(X, t) = \dot{u}(X, t)$ is the velocity. Equations (2.7) hold at all times t and for all X_α, X_β in $\mathscr{R}$. Equation $(2.7)_1$ is the one-dimensional form of the law of balance of momentum, equation $(2.7)_2$ is the one-dimensional form of Gauss' law for a charge free body, and equation $(2.7)_3$ is the one-dimensional form of the law of balance of energy. There should be no difficulty in verifying that equations (2.7) are but the one-dimensional forms of the more general equations recently derived by Tiersten[1]. When comparing equations (2.7) with Tiersten's equations it should be noted that the stress T as used here is the sum of the usual mechanical stress and the Maxwell stress occurring in Tiersten's work. In $(2.7)_3$, $-\Phi\dot{\mathscr{D}}$ is merely the one-dimensional form of the Poynting flux in a quasi-static electromagnetic field in which electrical effects predominate.

We assume that the function $\hat{\Gamma}(\cdot,\ \cdot,\ \cdot)$ is of class C^3; thus, by (2.2), $\hat{T}(\cdot,\ \cdot,\ \cdot)$, $\hat{\mathscr{D}}(\cdot,\ \cdot,\ \cdot)$, and $\hat{\theta}(\cdot,\ \cdot,\ \cdot)$ are of class C^2. We define the quantities α, β, γ, δ and κ by the relations

$$\alpha = \frac{\partial \hat{T}}{\partial \varepsilon}, \qquad \beta = \frac{\partial \hat{T}}{\partial \mathscr{E}} = -\frac{\partial \hat{\mathscr{D}}}{\partial \varepsilon}, \qquad \gamma = \frac{\partial \hat{T}}{\partial \eta} = \rho_0 \frac{\partial \hat{\theta}}{\partial \varepsilon},$$
$$\delta = -\frac{\partial \hat{\mathscr{D}}}{\partial \eta} = \rho_0 \frac{\partial \hat{\theta}}{\partial \mathscr{E}}, \qquad \kappa = \frac{\partial \hat{\mathscr{D}}}{\partial \mathscr{E}} \tag{2.8}$$

and we note that these quantities are of class C^1.

3. GENERAL PROPERTIES OF SHOCK WAVES†

We assume that the motion contains a shock wave moving with intrinsic velocity

$$U(t) = \frac{\mathrm{d}\,Y(t)}{\mathrm{d}t} > 0 \tag{3.1}$$

where $Y(t)$ is the material point at which the wave is to be found at time t. Thus, we assume that $u(\cdot,\ \cdot)$ and $\Phi(\cdot,\ \cdot)$ are continuous functions everywhere; while $\varepsilon(\cdot,\ \cdot)$, $v(\cdot,\ \cdot)$, $\mathscr{E}(\cdot,\ \cdot)$ and $\eta(\cdot,\ \cdot)$ have jump discontinuities across the shock wave they are continuous functions everywhere else. Hence it follows from (2.1) and (2.3) that $e(\cdot,\ \cdot)$, $T(\cdot,\ \cdot)$, $\mathscr{D}(\cdot,\ \cdot)$, $\theta(\cdot,\ \cdot)$, and $\Gamma(\cdot,\ \cdot)$ also have jump discontinuities across the shock wave.

In one-dimension, the kinematical condition of compatibility is

$$\frac{\mathrm{d}}{\mathrm{d}t}[f] = [\dot{f}] + U\left[\frac{\partial f}{\partial X}\right], \tag{3.2}$$

where $[f]$ denotes the jump in the function f, i.e. $[f] = f^- - f^+$ with $f^{\pm} = \lim f(X, t)$. Since $U(t) > 0$, f^- and f^+ are respectively, the limiting values of f immediately behind and just in front of the shock wave.

Equations (2.7) imply that for all $X \neq Y(t)$

$$\frac{\partial T}{\partial X} = \rho_0 \dot{v},$$
$$\frac{\partial \mathscr{D}}{\partial X} = 0, \tag{3.3}$$
$$\rho_0 \dot{\Gamma} = T\dot{\varepsilon} - \mathscr{D}\dot{\mathscr{E}},$$

and across the shock wave we have‡

† Recently, Chen and McCarthy [2] examined the behavior of shock waves propagating in elastic dielectrics within the context of the present theory but neglecting thermodynamic effects.

‡ We should always bear in mind that the Second Law requires $[\eta] \geq 0$.

$$[T] = -\rho_0 U[v], \quad \left[\frac{\partial T}{\partial X}\right] = \rho_0 [\dot{v}],$$
$$[\mathscr{D}] = 0, \qquad \left[\frac{\partial \mathscr{D}}{\partial X}\right] = 0, \tag{3.4}$$
$$-\rho_0 U\left[\Gamma + \frac{\mathscr{E}\mathscr{D}}{\rho_0} + \tfrac{1}{2}v^2\right] = [Tv] - [\Phi \dot{\mathscr{D}}], \quad \rho_0 [\dot{\Gamma}] = [T\dot{\varepsilon}] - [\mathscr{D}\dot{\mathscr{E}}].$$

On putting $f(\cdot, \cdot) = u(\cdot, \cdot)$ in (3.2) we find that

$$[v] = -U[\varepsilon] \tag{3.5}$$

and, on combining (3.4) and $(3.5)_1$, we obtain the classical result

$$U^2 = \frac{[T]}{\rho_0 [\varepsilon]} \tag{3.6}$$

for the intrinsic velocity of the shock. If we put $f(\cdot, \cdot) = v(\cdot, \cdot)$ and $f(\cdot, \cdot) = \varepsilon(\cdot, \cdot)$ consecutively in (3.2) and combine the resulting equations with $(3.4)_2$ and (3.5) we arrive at another standard relation†

$$2U\frac{\mathrm{d}[\varepsilon]}{\mathrm{d}t} + [\varepsilon]\frac{\mathrm{d}U}{\mathrm{d}t} = U^2\left[\frac{\partial \varepsilon}{\partial X}\right] - \frac{1}{\rho_0}\left[\frac{\partial T}{\partial X}\right]. \tag{3.7}$$

Once the constitutive laws of the material are known, equation (3.7) enables us to determine the shock amplitude equation of a shock propagating in that material.

Now, since $\Phi(\cdot, \cdot)$ is a continuous function everywhere,

$$[\Phi \dot{\mathscr{D}}] = \Phi[\dot{\mathscr{D}}]; \tag{3.8}$$

and on putting $f(\cdot, \cdot) = \mathscr{D}(\cdot, \cdot)$ in (3.2) and using $(3.4)_2$ and $(3.4)_3$ we conclude that $[\dot{\mathscr{D}}] = 0$. Thus, $(3.4)_5$ now becomes

$$-\rho_0 U\left[\Gamma + \frac{\mathscr{E}\mathscr{D}}{\rho_0} + \tfrac{1}{2}v^2\right] = [Tv], \tag{3.9}$$

which, together with $(3.4)_1$, (3.5), and (3.6), implies that

$$\rho_0[\Gamma] + \mathscr{D}[\mathscr{E}] - \tfrac{1}{2}(T^+ + T^-)[\varepsilon] = 0. \tag{3.10}$$

Equation (3.10) is a generalization of the well known Hugoniot relation to the case of non heat conducting elastic dielectrics. Our subsequent work will show that this equation and equation $(3.4)_3$ are of importance in the analysis of the basic properties of the shock transition.

To complete this Section, we note that it follows from (2.2,) $(3.4)_3$ and $(3.4)_6$ that

$$\dot{\eta} = 0 \tag{3.11}$$

and consequently

$$[\dot{\eta}] = 0. \tag{3.12}$$

† See Chen and Gurtin[3] and Achenbach and Herrmann[4].

4. THE SHOCK AMPLITUDE EQUATION

In this Section we use equation (3.7) to obtain an equation which governs the amplitude of a shock wave in a non heat conducting elastic dielectric.

It is clear from the definitions $(2.8)_1$ that, for all $X \neq Y(t)$, we have

$$\frac{\partial T}{\partial X} = \alpha \frac{\partial \varepsilon}{\partial X} + \beta \frac{\partial \mathscr{E}}{\partial X} + \gamma \frac{\partial \eta}{\partial X}. \tag{4.1}$$

Thus equations (3.7) and (4.1) imply that

$$2U \frac{\mathrm{d}[\varepsilon]}{\mathrm{d}t} + [\varepsilon] \frac{\mathrm{d}U}{\mathrm{d}t} = \left(U^2 - \frac{\alpha^-}{\rho_0}\right)\left[\frac{\partial \varepsilon}{\partial X}\right] - \frac{\beta^-}{\rho_0}\left[\frac{\partial \mathscr{E}}{\partial X}\right] - \frac{\gamma^-}{\rho_0}\left[\frac{\partial \eta}{\partial X}\right] - \frac{[\alpha]}{\rho_0}\left(\frac{\partial \varepsilon}{\partial X}\right)^+ - \frac{[\beta]}{\rho_0}\left(\frac{\partial \mathscr{E}}{\partial X}\right)^+ - \frac{[\gamma]}{\rho_0}\left(\frac{\partial \eta}{\partial X}\right)^+. \tag{4.2}$$

Here, and in what follows, we use the notation

$$f^+ = f(\varepsilon^+, \mathscr{E}^+, \eta^+). \qquad f^- = f(\varepsilon^- \ \mathscr{E}^-, \eta^-). \tag{4.3}$$

Again, the definitions (2.8) and $(3.3)_2$ imply that, for all $X \neq Y(t)$,

$$\kappa \frac{\partial \mathscr{E}}{\partial X} = \beta \frac{\partial \varepsilon}{\partial X} + \delta \frac{\partial \eta}{\partial X}, \tag{4.4}$$

and assuming that $\kappa(\varepsilon, \mathscr{E}, \eta) > 0$,

$$\left[\frac{\partial \mathscr{E}}{\partial X}\right] = \frac{\beta^-}{\kappa^-}\left[\frac{\partial \varepsilon}{\partial X}\right] + \frac{\delta^-}{\kappa^-}\left[\frac{\partial \eta}{\partial X}\right] + \left[\frac{\beta}{\kappa}\right]\left(\frac{\partial \varepsilon}{\partial X}\right)^+ + \left[\frac{\delta}{\kappa}\right]\left(\frac{\partial \eta}{\partial X}\right)^+. \tag{4.5}$$

Equation $(3.4)_6$ and the compatibility condition (3.2) with $f(\cdot, \cdot) = \varepsilon(\cdot, \cdot)$ yield the relation

$$\rho_0[\dot{\Gamma}] = T^- \frac{\mathrm{d}[\varepsilon]}{\mathrm{d}t} - UT^-\left[\frac{\partial \varepsilon}{\partial X}\right] + [T]\dot{\varepsilon}^+ - \mathscr{D}\frac{\mathrm{d}[\mathscr{E}]}{\mathrm{d}t} + U\mathscr{D}\left[\frac{\partial \mathscr{E}}{\partial X}\right]. \tag{4.6}$$

It follows from (2.2) that

$$\left[\frac{\partial \Gamma}{\partial X}\right] = \frac{T^-}{\rho_0}\left[\frac{\partial \varepsilon}{\partial X}\right] + \frac{1}{\rho_0}[T]\left(\frac{\partial \varepsilon}{\partial X}\right)^+ - \frac{\mathscr{D}}{\rho_0}\left[\frac{\partial \mathscr{E}}{\partial X}\right] + \theta^-\left[\frac{\partial \eta}{\partial X}\right] + [\theta]\left(\frac{\partial \eta}{\partial X}\right)^+. \tag{4.7}$$

Taking the d/dt derivative of the Hugoniot relation (3.10) and combining the result with the compatibility condition (3.2) with $f(\cdot, \cdot) = \Gamma(\cdot, \cdot)$, (4.6) and (4.7), we have

$$U\theta^-\left[\frac{\partial \eta}{\partial X}\right] = -\frac{[T]}{2\rho_0}\frac{\mathrm{d}[\varepsilon]}{\mathrm{d}t} - \frac{[T]}{\rho_0}\frac{\mathrm{d}\varepsilon^+}{\mathrm{d}t} - U[\theta]\left(\frac{\partial \eta}{\partial X}\right)^+ - \frac{[\mathscr{E}]\dot{\mathscr{D}}}{\rho_0} + \frac{[\varepsilon]}{2\rho_0}\left(\frac{\mathrm{d}T^+}{\mathrm{d}t} + \frac{\mathrm{d}T^-}{\mathrm{d}t}\right). \tag{4.8}$$

An elementary calculation, using the definitions (2.8) shows that

$$\frac{dT^+}{dt} + \frac{dT^-}{dt} = \alpha^- \frac{d[\varepsilon]}{dt} + \beta^- \frac{d[\mathscr{E}]}{dt} + U\gamma^- \left[\frac{\partial \eta}{\partial X}\right] + (\alpha^- + \alpha^+) \frac{d\varepsilon^+}{dt} + (\beta^- + \beta^+) \frac{d\mathscr{E}^+}{dt} + U(\gamma^- + \gamma^+)\left(\frac{\partial \eta}{\partial X}\right)^+ . \tag{4.9}$$

In view of equation $(3.4)_3$ we have

$$-\beta^- \frac{d[\varepsilon]}{dt} + \kappa^- \frac{d[\mathscr{E}]}{dt} - U\delta^- \left[\frac{\partial \eta}{\partial X}\right] - [\beta] \frac{d\varepsilon^+}{dt} + [\kappa] \frac{d\mathscr{E}^+}{dt} + U[\delta] \left(\frac{d\eta}{\partial X}\right)^+ = 0, \tag{4.10}$$

so that

$$\frac{d[\mathscr{E}]}{dt} = \frac{\beta^-}{\kappa^-} \frac{d[\varepsilon]}{dt} + \frac{U\delta^-}{\kappa^-} \left[\frac{\partial \eta}{\partial X}\right] + \frac{[\beta]}{\kappa^-} \dot{\varepsilon}^+ - \frac{[\kappa]}{\kappa^-} \dot{\mathscr{E}}^+ + U\left(\frac{[\beta]}{\kappa^-} - \frac{\beta^+}{\kappa^- \kappa^+} [\kappa]\right)\left(\frac{\partial \varepsilon}{\partial X}\right)^+ + U\left[\frac{\delta}{\kappa}\right]\left(\frac{\partial \eta}{\partial X}\right)^+ . \tag{4.11}$$

In deriving (4.11) we have used equation (4.4). Equations (4.8,) (4.9) and (4.11) imply

$$\begin{aligned}\left[\frac{\partial \eta}{\partial X}\right] = {} & \frac{H^-(1-\xi)}{G^- U(2\tau - 1)} \frac{d[\varepsilon]}{dt} + \frac{1}{G^- U(2\tau - 1)} \{2H^-(1-\xi) - [H] - \beta^+ N\}\dot{\varepsilon}^+ \\ & + \frac{1}{G^-(2\tau - 1)} \{2H^-(1-\xi) - [H]\}\left(\frac{\partial \varepsilon}{\partial X}\right)^+ \\ & + \frac{1}{G^-(2\tau - 1)}\left\{G^- + G^+ - 2\rho_0 \frac{[\theta]}{[\varepsilon]}\right\}\left(\frac{\partial \eta}{\partial X}\right)^+ \\ & + \frac{\kappa^+ N}{G^- U(2\tau - 1)} \dot{\mathscr{E}}^+\end{aligned} \tag{4.12}$$

where

$$H = \alpha + \frac{\beta^2}{\kappa}, \qquad G = \gamma + \frac{\beta\delta}{\kappa},$$
$$L = \frac{\beta}{\kappa}, \qquad N = L^- + L^+ - 2\frac{[\mathscr{E}]}{[\varepsilon]} \tag{4.13}$$

and

$$\tau = \frac{\rho_0 \theta^-}{G^-[\varepsilon]}, \qquad \xi = \frac{\rho_0 U^2}{H^-} . \tag{4.14}$$

Differentiating the formula for the intrinsic speed (3.6), we have

$$2\rho_0 U[\varepsilon] \frac{dU}{dt} = \frac{d[T]}{dt} - \rho_0 U^2 \frac{d[\varepsilon]}{dt}, \tag{4.15}$$

which, together with (2.8) and (4.12), yields

$$
\begin{aligned}
\frac{dU}{dt} = {} & \frac{\tau H^-(1-\xi)}{\rho_0 U(2\tau-1)[\varepsilon]}\frac{d[\varepsilon]}{dt} \\
& + \frac{1}{\rho_0[\varepsilon](2\tau-1)}\left\{\left\{H^-(1-\xi)+(\tau-1)[H]-\tfrac{1}{2}\beta^+N-(\tau-\tfrac{1}{2})\beta^+[L]\right\}\frac{\dot{\varepsilon}^+}{U}\right. \\
& + \left\{H^-(1-\xi)+(\tau-1)[H]\right\}\left(\frac{\partial\varepsilon}{\partial X}\right)^+ \\
& + \left\{\tfrac{1}{2}(G^-+G^+)+(\tau-\tfrac{1}{2})[G]-\frac{\rho_0[\theta]}{[\varepsilon]}\right\}\left(\frac{\partial\eta}{\partial X}\right)^+ \\
& \left. + \left\{\tfrac{1}{2}\kappa^+N+(\tau-\tfrac{1}{2})\kappa^+[L]\right\}\frac{\dot{\varepsilon}^+}{U}\right\}.
\end{aligned}
\tag{4.16}
$$

Finally, substituting (4.4), (4.5), (4.12) and (4.16) into (4.2), we have:

Theorem 1

The amplitude of a shock wave propagating in an elastic dielectric material which does not conduct heat satisfies the equation

$$
\frac{d[\varepsilon]}{dt} = \frac{U(1-\xi)(2\tau-1)}{(3\xi+1)\tau-(3\xi-1)}\left\{\lambda_c - \left[\frac{\partial\varepsilon}{\partial X}\right]\right\}, \tag{4.17}\dagger
$$

where

$$
\begin{aligned}
\lambda_c = {} & -\frac{1}{H^-(1-\xi)(2\tau-1)}\left\{\left\{3H^-(1-\xi)+(\tau-2)[H]-\tfrac{3}{2}\beta^+N-(\tau-\tfrac{1}{2})\beta^+[L]\right\}\frac{\dot{\varepsilon}^+}{U}\right. \\
& + \left\{3H^-(1-\xi)+3(\tau-1)[H]\right\}\left(\frac{\partial\varepsilon}{\partial X}\right)^+ \\
& + 3\left\{(\tau-\tfrac{1}{2})[G]+\tfrac{1}{2}(G^++G^-)-\rho_0\frac{[\theta]}{[\varepsilon]}\right\}\left(\frac{\partial\eta}{\partial X}\right)^+ \\
& \left. + \left\{\tfrac{3}{2}\kappa^+N+(\tau-\tfrac{1}{2})\kappa^+[L]\right\}\frac{\dot{\mathscr{E}}^+}{U}\right\}.
\end{aligned}
\tag{4.18}
$$

Clearly, equation (4.17) is extremely complicated and we can only hope to deduce useful information from it by adopting additional assumptions concerning the properties of the material and the nature of the shock wave under examination. We return to the study of equation (4.17) in the following section.

† Even though (4.17) is of the same form as those arising from numerous other theories (see Chen and Gurtin[5–7], and Chen[8]), a cursory examination reveals that the similarity is superficial.

5. PROPERTIES OF SHOCK TRANSITION

In view of our assumption that $\kappa(\varepsilon, \mathscr{E}, \eta) > 0$, it follows from $(2.1)_3$ that there exists a function $\tilde{\mathscr{E}}(\cdot, \cdot, \cdot)$ such that

$$\mathscr{E} = \tilde{\mathscr{E}}(\varepsilon, \mathscr{D}, \eta). \tag{5.1}$$

We define the functions $\tilde{\Gamma}(\cdot, \cdot, \cdot)$, $\tilde{T}(\cdot, \cdot, \cdot)$, and $\tilde{\theta}(\cdot, \cdot, \cdot)$ as follows

$$\begin{aligned} \Gamma &= \tilde{\Gamma}(\varepsilon, \mathscr{D}, \eta) = \hat{\Gamma}(\varepsilon, \tilde{\mathscr{E}}(\varepsilon, \mathscr{D}, \eta), \eta), \\ T &= \tilde{T}(\varepsilon, \mathscr{D}, \eta) = \hat{T}(\varepsilon, \tilde{\mathscr{E}}(\varepsilon, \mathscr{D}, \eta), \eta), \\ \theta &= \tilde{\theta}(\varepsilon, \mathscr{D}, \eta) = \hat{\theta}(\varepsilon, \tilde{\mathscr{E}}(\varepsilon, \mathscr{D}, \eta), \eta). \end{aligned} \tag{5.2}$$

It follows from $(3.3)_2$, (4.4) and (5.1) that

$$\frac{\partial \tilde{\mathscr{E}}}{\partial \varepsilon} = \frac{\beta}{\kappa}, \qquad \frac{\partial \tilde{\mathscr{E}}}{\partial \mathscr{D}} = \frac{1}{\kappa}, \qquad \frac{\partial \tilde{\mathscr{E}}}{\partial \eta} = \frac{\delta}{\kappa}, \tag{5.3}$$

and, on using (2.8), (4.13), (5.2) and (5.3), we find that

$$\begin{aligned} \frac{\partial \tilde{T}}{\partial \varepsilon} &= \frac{\partial \hat{T}}{\partial \varepsilon} + \frac{\partial \hat{T}}{\partial \mathscr{E}} \frac{\partial \tilde{\mathscr{E}}}{\partial \varepsilon} = \alpha + \frac{\beta^2}{\kappa} = H, \\ \frac{\partial \tilde{T}}{\partial \eta} &= \frac{\partial \hat{T}}{\partial \eta} + \frac{\partial \hat{T}}{\partial \mathscr{E}} \frac{\partial \tilde{\mathscr{E}}}{\partial \eta} = \gamma + \frac{\beta\delta}{\kappa} = G. \end{aligned} \tag{5.4}$$

We assume that

$$H(\varepsilon, \mathscr{D}, \eta) > 0 \quad \text{and} \quad G(\varepsilon, \mathscr{D}, \eta) \neq 0. \tag{5.5}$$

It follows from $(5.2)_2$ and $(5.5)_2$ that we can write

$$\eta = \tilde{T}^{-1}(\varepsilon, \mathscr{D}, \eta). \tag{5.6}$$

At a given instant the values of ε^+, $\mathscr{E}^+$, and η^+ just ahead of the shock wave are fixed, and in view of $(2.1)_3$ and $(3.4)_3$ so is the value of $\mathscr{D}$. It follows that the thermodynamic state immediately behind the shock wave is determined once ε^- and T^- are known. Of course, ε^- and T^- cannot be arbitrary for they must satisfy the relation (3.10) which we now write in the form

$$\begin{aligned} &\rho_0 \tilde{\Gamma}(\varepsilon^-, \mathscr{D}, \tilde{T}^{-1}(\varepsilon^-, \mathscr{D}, T^-)) - \rho_0 \tilde{\Gamma}(\varepsilon^+, \mathscr{D}, \tilde{T}^{-1}(\varepsilon^+, \mathscr{D}, T^+)) \\ &\quad + \mathscr{D}\tilde{\mathscr{E}}(\varepsilon^-, \mathscr{D}, \tilde{T}^{-1}(\varepsilon^-, \mathscr{D}, T^-)) - \mathscr{D}\tilde{\mathscr{E}}(\varepsilon^+, \mathscr{D}, \tilde{T}^{-1}(\varepsilon^+, \mathscr{D}, T^+)) - \tfrac{1}{2}(T^+ + T^-)[\varepsilon] = 0. \end{aligned} \tag{5.7}$$

As is usual in shock wave studies, we assume that in the (ε^-, T^-) plane the relation (5.7) can be represented by a curve $T^- = T_H(\varepsilon^-)$. The function T_H also depends on ε^+, η^+, and $\mathscr{D}$ or alternatively, on ε^+, η^+, and $\mathscr{E}^+$.

The foregoing assumption enables us to prove that if in addition to $(5.5)_1$ we assume that

$$\frac{\partial^2 \tilde{T}(\varepsilon, \mathscr{D}, \eta)}{\partial \varepsilon^2} < 0, \qquad \text{for all} \qquad (\varepsilon, \mathscr{D}, \eta),\ \varepsilon \leq 0, \tag{5.8}$$

then

(i) the wave is a compressive shock, i.e. $[\varepsilon] < 0$,

(ii) the intrinsic speed of the shock is supersonic with respect to the material ahead of the shock and subsonic with respect to the material behind it, i.e.

$$H^+ < \rho_0 U^2 < H^-, \tag{5.9}$$

(iii) the entropy η^- immediately behind the shock increases with decreasing ε^-.

If instead of (5.8) we assume that

$$\frac{\partial^2 \hat{T}(\varepsilon, \mathscr{D}, \eta)}{\partial \varepsilon^2} > 0, \quad \text{for all} \quad (\varepsilon, \mathscr{D}, \eta), \varepsilon \geq 0, \tag{5.10}$$

then (ii) of the previous paragraph still holds, but instead of (i) and (iii) we have

(i*) the wave is an expansion shock, i.e. $[\varepsilon] > 0$,

(iii*) the entropy η^- immediately behind the shock increases with increasing ε^-.

The derivation of the foregoing results follows closely the classical arguments given for elastic fluids.† Since the proof of these results is straightforward, but involves rather lengthy algebra we omit the details. Of course, results of this type have been given in a number of earlier studies.

Now (5.1), (5.6) and our assumptions concerning the Hugoniot relation (5.7) imply that there exist functions $\mathscr{E}_H(\cdot, \cdot, \cdot, \cdot)$ and $\eta_H(\cdot, \cdot, \cdot, \cdot)$ such that

$$\begin{aligned} [\mathscr{E}] &= \mathscr{E}_H([\varepsilon], \varepsilon^+, \mathscr{E}^+, \eta^+), \\ [\eta] &= \eta_H([\varepsilon], \varepsilon^+, \mathscr{E}^+, \eta^+). \end{aligned} \tag{5.11}$$

While we do not know the explicit forms of the functions defined by (5.11) we can determine their derivatives. Indeed, taking the d/dt derivative of $(5.11)_2$ and using (3.2) with $f(\cdot, \cdot) = \eta(\cdot, \cdot)$, (3.12), (4.4) and (4.12), we have

$$\begin{aligned} \frac{\partial \eta_H}{\partial [\varepsilon]} &= \frac{H^-(1-\xi)}{G^-(2\tau - 1)}, \\ \frac{\partial \eta_H}{\partial \varepsilon^+} &= \left\{2 - \frac{[H]}{H^-(1-\xi)} - \frac{\beta^+ N}{H^-(1-\xi)}\right\} \frac{\partial \eta_H}{\partial [\varepsilon]}, \\ \frac{\partial \eta_H}{\partial \mathscr{E}^+} &= \frac{\kappa^+ N}{G^-(2\tau - 1)}, \\ \frac{\partial \eta_H}{\partial \eta^+} &= \frac{1}{H^-(1-\xi)} \left\{G^- + G^+ - 2\rho_0 \frac{[\theta]}{[\varepsilon]} - \delta^+ N\right\} \frac{\partial \eta_H}{\partial [\varepsilon]}. \end{aligned} \tag{5.12}$$

Likewise, differentiation of $(5.11)_1$ yields

$$\begin{aligned} \frac{\partial \mathscr{E}_H}{\partial [\varepsilon]} &= \frac{\delta^-}{\kappa^-} \frac{\partial \eta_H}{\partial [\varepsilon]} + \frac{\beta^-}{\kappa^-}, \\ \frac{\partial \mathscr{E}_H}{\partial \varepsilon^+} &= \frac{\delta^-}{\kappa^-} \left\{2 - \frac{[H]}{H^-(1-\xi)} - \frac{\beta^+ N}{H^-(1-\xi)}\right\} \frac{\partial \eta_H}{\partial [\varepsilon]} + \frac{[\beta]}{\kappa^-}, \\ \frac{\partial \mathscr{E}_H}{\partial \mathscr{E}^+} &= \frac{\delta^- \kappa^+ N}{\kappa^- G^-(2\tau - 1)} - \frac{[\kappa]}{\kappa^-}, \\ \frac{\partial \mathscr{E}_H}{\partial \eta^+} &= \frac{\delta^-}{\kappa^- H^-(1-\xi)} \left\{G^- + G^+ - 2\rho_0 \frac{[\theta]}{[\varepsilon]} - \delta^+ N\right\} \frac{\partial \eta_H}{\partial [\varepsilon]} + \left[\frac{\delta}{\kappa}\right] - \delta^+ \left[\frac{1}{\kappa}\right]. \end{aligned} \tag{5.13}$$

† See Weyl[9].

Henceforth, let us consider a compression shock propagating in a body which is initially in compression, i.e.†

$$[\varepsilon] < 0, \qquad \varepsilon^+ \leq 0. \tag{5.14}$$

In view of our preceding results we see that ξ, defined by $(4.14)_2$, must satisfy the inequality

$$0 < \xi < 1, \tag{5.15}$$

and that

$$\frac{H^-(1-\xi)}{G^-(2\tau - 1)} < 0. \tag{5.16}$$

Thus, by (5.5), (5.15) and (5.16), we see that τ, defined by $(4.14)_1$, obeys the relations

$$\begin{aligned} G^- < 0 &\Leftrightarrow \tau > \tfrac{1}{2}, \\ G^- > 0 &\Leftrightarrow \tau < 0. \end{aligned} \tag{5.17}$$

With the results (5.15) and (5.17) we may now state the implications of the shock amplitude equation (4.17) on the behavior of the compression shock, defined by (5.14). Indeed, we have the following:

(i) *If* $G^- < 0$, *or if* $G^- > 0$ *and* $\tau < \dfrac{3\xi - 1}{3\xi + 1}$, *then at any instant*

$$\left[\frac{\partial \varepsilon}{\partial X}\right] < \lambda_c \Leftrightarrow \frac{\mathrm{d}|[\varepsilon]|}{\mathrm{d}t} < 0,$$

$$\left[\frac{\partial \varepsilon}{\partial X}\right] > \lambda_c \Leftrightarrow \frac{\mathrm{d}|[\varepsilon]|}{\mathrm{d}t} > 0.$$

(ii) *If* $G^- > 0$ *and* $\tau > \dfrac{3\xi - 1}{3\xi + 1}$, *then at any instant*

$$\left[\frac{\partial \varepsilon}{\partial X}\right] < \lambda_c \Leftrightarrow \frac{\mathrm{d}|[\varepsilon]|}{\mathrm{d}t} > 0,$$

$$\left[\frac{\partial \varepsilon}{\partial X}\right] > \lambda_c \Leftrightarrow \frac{\mathrm{d}|[\varepsilon]|}{\mathrm{d}t} < 0.$$

(iii) *Whenever* $G^- < 0$ *or* $G^- > 0$, *we have at any instant*

$$\left[\frac{\partial \varepsilon}{\partial X}\right] = \lambda_c \Leftrightarrow \frac{\mathrm{d}|[\varepsilon]|}{\mathrm{d}t} = 0.$$

In view of the preceding results we call λ_c, defined by (4.18), the critical jump in strain gradient. It is a generalization to thermoelastic dielectrics of those which arise in a number of other theories.‡

† Results corresponding to those presented in the remainder of this section and Section 6 can be readily established for an expansion shock.

‡ See Chen and Gurtin[5–7], and Chen[8].

Now, let us record certain special cases of λ_c which are of interest. First, if the region ahead of the shock is steady, then by $(2.2)_1$, (2.8), (3.3) and (4.4) it can be shown that λ_c reduces to

$$\lambda_c = -\frac{3}{(2\tau-1)(1-\xi)}\left\{\tau - \xi - \rho_0 \frac{\theta^+}{G^+[\varepsilon]}\frac{H^+}{H^-}\right\}\left(\frac{\mathrm{d}\varepsilon}{\mathrm{d}X}\right)^+, \tag{5.18}$$

On the other hand, if the region ahead of the shock is not only steady but also uniform, then λ_c vanishes. In which case, we have

$$\begin{aligned}\frac{\mathrm{d}[\eta]}{\mathrm{d}t} &= \frac{H^-(1-\xi)}{G^-(2\tau-1)}\frac{\mathrm{d}[\varepsilon]}{\mathrm{d}t},\\ \frac{\mathrm{d}[\mathscr{E}]}{\mathrm{d}t} &= \left\{\frac{\delta^- H^-(1-\xi)}{\kappa^- G^-(2\tau-1)} + \frac{\beta^-}{\kappa^-}\right\}\frac{\mathrm{d}[\varepsilon]}{\mathrm{d}t}.\end{aligned} \tag{5.19}$$

6. PROPERTIES OF TEMPERATURE AND ELECTRIC FIELD DURING SHOCK TRANSITION

We define the specific heat by the relation

$$c = \hat{c}(\varepsilon, \mathscr{E}, \eta) = \hat{\theta}(\varepsilon, \mathscr{E}, \eta)\left(\frac{\partial\hat{\theta}(\varepsilon, \mathscr{E}, \eta)}{\partial\eta}\right)^{-1} \tag{6.1}$$

and assume that

$$\hat{c}(\varepsilon, \mathscr{E}, \eta) > 0. \tag{6.2}$$

By $(2.1)_4$ and (5.6) we have

$$[\theta] = \hat{\theta}(\varepsilon^+ + [\varepsilon], \mathscr{E}^+ + \mathscr{E}_H([\varepsilon], \varepsilon^+, \mathscr{E}^+, \eta^+), \eta^+ + \eta_H([\varepsilon], \varepsilon^+, \mathscr{E}^+, \eta^+)) - \hat{\theta}(\varepsilon^+, \mathscr{E}^+, \eta^+). \tag{6.3}$$

Differentiating (6.3) with respect to $[\varepsilon]$ and using (2.8), $(5.12)_1$, $(5.13)_1$ and (6.1), we find that

$$\frac{\partial[\theta]}{\partial[\varepsilon]} = \frac{G^-}{\rho_0} + \left(\frac{\theta^-}{c^-} + \frac{(\delta^-)^2}{\rho_0\kappa^-}\right)\frac{\partial\eta_H}{\partial[\varepsilon]}. \tag{6.4}$$

In view of our earlier assumptions the coefficient of $\partial\eta_H/\partial[\varepsilon]$ in (6.4) is positive.

Consider a compressive shock wave ($[\varepsilon] < 0$) which is propagating into a region which is in a state of compression ($\varepsilon^+ \leq 0$). Thus, by $(5.12)_1$, (5.16) and (6.4), we have the following:

(i) If $G^- < 0$, then

$$\frac{\partial[\theta]}{\partial[\varepsilon]} < 0. \tag{6.5}$$

(ii) If $G^- > 0$, then

$$\frac{\partial[\theta]}{\partial[\varepsilon]} < 0 \text{ whenever } G^- < -\left(\rho_0\frac{\theta^-}{c^-} + \frac{(\delta^-)^2}{\kappa^-}\right)\cdot\frac{\partial\eta_H([\varepsilon], \varepsilon^+, \mathscr{E}^+, \eta^+)}{\partial[\varepsilon]} \tag{6.6}$$

or

$$\frac{\partial[\theta]}{\partial[\varepsilon]} > 0 \text{ whenever } G^- > -\left(\rho_0\frac{\theta^-}{c^-} + \frac{(\delta^-)^2}{\kappa^-}\right)\frac{\partial\eta_H([\varepsilon], \varepsilon^+, \mathscr{E}^+, \eta^+)}{\partial[\varepsilon]}. \tag{6.7}$$

Further, in view of $(5.12)_1$, $(5.13)_1$ and (5.16), we have the following:

(i*) If $\beta^- < 0$ and $\delta^- > 0$, then

$$\frac{\partial[\mathscr{E}]}{\partial[\varepsilon]} < 0. \tag{6.8}$$

(ii*) If $\beta^- < 0$ and $\delta^- < 0$, then

$$\frac{\partial[\mathscr{E}]}{\partial[\varepsilon]} < 0 \text{ whenever } \beta^- < -\delta^- \frac{\partial\eta_H([\varepsilon], \varepsilon^+, \mathscr{E}^+, \eta^+)}{\partial[\varepsilon]}, \tag{6.9}$$

or

$$\frac{\partial[\mathscr{E}]}{\partial[\varepsilon]} > 0 \text{ whenever } \beta^- > -\delta^- \frac{\partial\eta_H([\varepsilon], \varepsilon^+, \mathscr{E}^+, \eta^+)}{\partial[\varepsilon]}. \tag{6.10}$$

(iii*) If $\beta^- > 0$ and $\delta^- < 0$, then

$$\frac{\partial[\mathscr{E}]}{\partial[\varepsilon]} > 0. \tag{6.11}$$

(iv*) If $\beta^- > 0$ and $\delta^- > 0$, then

$$\frac{\partial[\mathscr{E}]}{\partial[\varepsilon]} < 0 \text{ whenever } \beta^- < -\delta^- \frac{\partial\eta_H([\varepsilon], \varepsilon^+, \mathscr{E}^+, \eta^+)}{\partial[\varepsilon]}, \tag{6.12}$$

or

$$\frac{\partial[\mathscr{E}]}{\partial[\varepsilon]} > 0 \text{ whenever } \beta^- > -\delta^- \frac{\partial\eta_H([\varepsilon], \varepsilon^+, \mathscr{E}^+, \eta^+)}{\partial[\varepsilon]}. \tag{6.13}$$

Equations (6.5)–(6.13) give the conditions when the temperature and electric field increases or decreases across the shock. In view of (6.5) we see that for most materials the temperature behind the shock increases monotonically with decreasing ε^-. On the other hand, the general behavior of the electric field across the shock is uncertain.

7. INFINITESIMAL SHOCK WAVES

Here we determine some of the properties of the shock in the limit as $\varepsilon^- \to \varepsilon^+$; that is, we consider the case of a shock of infinitesimal amplitude ($|[\varepsilon]| \ll 1$). In particular, we derive approximate expressions for $[\mathscr{E}]$, $[T]$, $[\theta]$, $[e]$ and U for such a shock. Henceforth, we assume that $\hat{\Gamma}(\cdot, \cdot, \cdot)$ is of class C^4.

To begin with, we have the following identities:

$$\begin{aligned}
H^* &= \frac{\partial H}{\partial \varepsilon} = \alpha^* + 3\frac{\beta^2}{\kappa^2}\beta^* - \frac{\beta^3}{\kappa^3}\kappa^* + 3\frac{\beta}{\kappa}\beta^\dagger,\\
H^\dagger &= \frac{\partial H}{\partial \mathscr{E}} = \beta^\dagger + 2\frac{\beta}{\kappa}\beta^* - \frac{\beta^2}{\kappa^2}\kappa^*,\\
G^* &= \frac{\partial^2 \tilde{\theta}}{\partial \varepsilon^2} = \frac{1}{\rho_0}\left\{\gamma^* + \frac{2\beta}{\kappa}\gamma^\dagger + \frac{\beta^2}{\kappa^2}\delta^* + \frac{\gamma}{\kappa}\left(\beta^\dagger + 2\frac{\beta}{\kappa}\beta^* - \frac{\beta^2}{\kappa^2}\kappa^*\right)\right\},
\end{aligned} \tag{7.1}$$

where

$$\alpha^* = \frac{\partial^2 \hat{T}}{\partial \varepsilon^2}, \qquad \beta^* = \frac{\partial^2 \hat{T}}{\partial \mathscr{E}^2}, \qquad \kappa^* = \frac{\partial^2 \hat{\mathscr{D}}}{\partial \mathscr{E}^2}, \qquad \beta\dagger = \frac{\partial^2 \hat{T}}{\partial \varepsilon\, \partial \mathscr{E}},$$
$$\gamma^* = \rho_0 \frac{\partial^2 \hat{\theta}}{\partial \varepsilon^2}, \qquad \delta^* = \rho_0 \frac{\partial^2 \hat{\theta}}{\partial \mathscr{E}^2}, \qquad \gamma\dagger = \rho_0 \frac{\partial^2 \hat{\theta}}{\partial \varepsilon\, \partial \mathscr{E}}. \tag{7.2}$$

Employing a standard argument, it follows from (5.7), (2.8) and $(7.1)_1$ that

$$[\eta] = \tfrac{1}{12} H_0^*[\varepsilon]^3 + o([\varepsilon]^3), \tag{7.3}$$

where $H_0^* = H^*|_{[\varepsilon]=0}$. Now

$$[\mathscr{E}] = \left.\frac{\partial \mathscr{E}_H}{\partial[\varepsilon]}\right|_{[\varepsilon]=0} [\varepsilon] + \tfrac{1}{2} \left.\frac{\partial^2 \mathscr{E}_H}{\partial [\varepsilon]^2}\right|_{[\varepsilon]=0} [\varepsilon]^2 + o([\varepsilon]^2). \tag{7.4}$$

By $(5.13)_1$, $(7.1)_2$ and (7.3), we have

$$\left.\frac{\partial \mathscr{E}_H}{\partial[\varepsilon]}\right|_{[\varepsilon]=0} = \frac{\beta^+}{\kappa^+}, \qquad \left.\frac{\partial^2 \mathscr{E}_H}{\partial[\varepsilon]^2}\right|_{[\varepsilon]=0} = \frac{H_0^\dagger}{\kappa^+}. \tag{7.5}$$

Thus, we have

$$[\mathscr{E}] = \frac{\beta^+}{\kappa^+}[\varepsilon] + \frac{H_0^\dagger}{2\kappa^+}[\varepsilon]^2 + o([\varepsilon]^2). \tag{7.6}$$

Similar arguments may be used to show that

$$\begin{aligned}
[T] &= H^+[\varepsilon] + \tfrac{1}{2}H_0^*[\varepsilon]^2 + o([\varepsilon]^2),\\
[\theta] &= \frac{1}{\rho_0} G^+[\varepsilon] + \tfrac{1}{2}G_0^*[\varepsilon]^2 + o([\varepsilon]^2),\\
[e] &= T^+[\varepsilon] + \frac{1}{2}\left(H^+ - \frac{\varepsilon_0}{2\rho_0}\left(\frac{\beta^+}{\kappa^+}\right)^2\right)[\varepsilon]^2 + o([\varepsilon]^2),\\
U^2 &= U_0^2 + \frac{H_0^*}{2\rho_0}[\varepsilon] + o([\varepsilon]),
\end{aligned} \tag{7.7}$$

where

$$U_0^2 = \frac{H^+}{\rho_0} = \{\alpha^+ + (\beta^+)^2/\kappa^+\}/\rho_0. \tag{7.8}$$

These results should prove to be quite useful in practice when the amplitude of the shock is small.

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REFERENCES

1. H. F. Tiersten, On the Nonlinear Equations of Thermoelectroelasticity, *Int. J. Engng Sci.* **9**, 587–604 (1971).
2. P. J. Chen and M. F. McCarthy, One-Dimensional Shock Waves in Elastic Dielectrics, Instituto Lombardo di Scienze, Rendiconti, **107**, 715–727 (1973).
3. P. J. Chen and M. E. Gurtin, On the Growth of One-Dimensional Shock Waves in Materials with Memory, *Arch. Rational Mech. Anal.* **36**, 33–46 (1970).
4. J. D. Achenbach and G. Herrmann, Propagation of Second-order Thermo-mechanical Disturbances in Viscoelastic Solids, *Procs. IUTAM Symp. East Kilbride*, (June 25–28, 1968), edited by B. A. Boley. Springer, Vienna–New York (1970).
5. P. J. Chen and M. E. Gurtin, The Growth of One-dimensional Shock Waves in Elastic Non-conductors, *Int. J. Solids Struct.* **7**, 5–10 (1971).
6. P. J. Chen and M. E. Gurtin, Growth and Decay of One-dimensional Shock Waves in Fluids with Internal State Variables, *Phys. Fluids*, **14**, 1091–1094 (1971).
7. P. J. Chen and M. E. Gurtin, Thermodynamic Influences on the Growth of One-dimensional Shock Waves in Materials with Memory, *Zeit. angew. Math. Phys.* **23**, 69–79 (1972).
8. P. J. Chen, Growth and Decay of Waves in Solids, *Handbuch der Physik*, **VIa**,/3, edited by C. Truesdell. Springer, Berlin (1973).
9. H. Weyl, Shock Waves in Arbitrary Fluids, *Comm. Pure Appl. Math.* **2**, 103–122 (1949).

Абстракт — В работе исследуется поведение одномерных ударных волн, распространяющихся в упругих диэлектриках. Принимается во внимание наличие термодинамических эффектов, но пренебрегается возможность теплопроводности. После вывоза дифференциального уравнения, которому амплитуда удара должна удовлетворять, исследуется свойства перехода удара. В особенности, находится, что классические результаты перехода удара можно обобщить для настоящей ситуации. Эти результаты, по очереди, разрешают вычитать причастность уравнения амплитуды удара на поведение удара. Находится что критерия роста или затухания удара зависит от относительных величин скачка в градиенте деформации и λ_c, названного критическим скачком в градиенте деформации. Это, очевидно, ожидается. Обсуждаются, также, свойства поля температуры и электрического поля во время перехода удара. Определяются частные результаты, важны для случая инфинитезимальной амплитуды удара.